UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF PURE AND APPLIED MATHEMATICS

MODULE: MATHEMATICAL ANALYSIS B – MAA00B1

CAMPUS: APK

ASSESSMENT: EXAMINATION

DATE: 27 NOVEMBER 2017

ASSESSORS: MR W VAN REENEN

INTERNAL MODERATOR: MR M POTGIETER

DURATION: 2 HOURS

89

INITIALS AND SURNAME: ____________________

STUDENT NUMBER: ____________________

CONTACT NUMBER: ____________________

NUMBER OF PAGES: 14 (INCLUDING COVER PAGE)

INSTRUCTIONS:
- ANSWER ALL THE QUESTIONS IN PEN.
- ALL GRAPHS MUST BE DRAWN IN PEN.
- NO PENCIL OR TIPEX ALLOWED.
- SHOW ALL THE NECESSARY CALCULATIONS CLEARLY.
- IF FORMULAS ARE USED THEY MUST BE STATED AS MARKS ARE GIVEN TO THEM.
- SCIENTIFIC AND FINANCIAL CALCULATORS ARE ALLOWED.
- IF NECESSARY, ROUND OFF TO TWO DECIMAL PLACES.
- THE QUESTIONS CAN BE ANSWERED IN ANY ORDER.

Question 1 **[3]**

Given that $f(x) = e^{2x+2}$, determine $f^{-1}(x)$.

Question 2 **[6]**

Given the following matrices:

$$A = \begin{bmatrix} 2 & -1 \\ 2 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix} \qquad C = \begin{bmatrix} -1 & 3 \\ 5 & -1 \end{bmatrix} \qquad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

2.1 Determine: $A + 5I$ [2]

2.2 Determine: $A^T - \frac{1}{2}C$ [2]

2.3 Determine: B^3 [2]

Question 3 **[5]**

Find A^{-1} if $A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$

Question 4 **[9]**

Given the input-output matrix $M = \begin{bmatrix} 200 & 500 & 500 \\ 400 & 200 & 900 \\ 600 & 800 & 0 \end{bmatrix}$

4.1 State the internal demand matric C. [1]

4.2 State the external demand matrix D. [1]

4.3 Determine the Leontief matrix A. [1]

4.4 Determine the production matrix X. [1]

4.5 If the external demand changes to $D_{new} = \begin{bmatrix} 600 \\ 805 \end{bmatrix}$, determine the new production matrix X_{new}. (Hint: $(I - A)X = D$)) [5]

Question 5 **[5]**

Differentiate the following function through first principles: $f(x) = \frac{x+3}{x-3}$

{Note: You are not allowed to use the differentiation rules}

Question 6 **[8]**

Given the following case-defined function:

$$f(x) = \begin{cases} (x+1)^2 - 1 & \text{if} \quad -3 \leq x < 1 \\ 3 & \text{if} \quad 1 < x < 3 \\ -x + 6 & \text{if} \quad 3 \leq x \leq 6 \end{cases}$$

6.1 Graph the function. [3]

6.2 Determine:

6.2.1 $\lim_{x \to 1^-} f(x)$. [1]

6.2.2 $\lim_{x \to 1^+} f(x)$. [1]

6.2.3 $\lim_{x \to 1} f(x)$. [1]

6.2.4 $f(1)$. [1]

6.3 Is the case defined function continuous at $x = 1$? (**YES** or **NO**) [1]

Question 7 **[2]**

Determine the following limits:

7.1 $\lim_{x \to -2} \frac{x^4 - 16}{x + 2}$ [1]

7.2 $\lim_{x \to -\infty} \frac{8x^4 - 7x^3 - 5x^2 - 2x}{4x^4 + 1000}$ [1]

Question 8 **[10]**

Differentiate the following functions. You do **NOT** have to simplify your answers.

8.1 $y = \ln \pi^{\log e}$ [1]

8.2 $y = \sqrt[4]{\sqrt[3]{\sqrt{x}}}$ [1]

8.3 $y = \ln\sqrt{3x - 3}$ [1.5]

8.4 $y = 4e^{x^3 + x^2 + x}$ [1.5]

8.5 $y = \sqrt[3]{5x - 5}$ [2]

8.6 $y = \frac{e^x}{\ln x}$ [3]

Question 9 **[4]**

Use implicit differentiation to find the derivative:

$$\ln x \cdot e^{y} + 2x^{2} + 2y^{2} = 2$$

Question 10 **[5]**

Prove the following differentiation rule:

If $f(x) = m(x).n(x)$, then $f'(x) = n(x).m'(x) + m(x).n'(x)$.

Question 11 **[5]**

Use logarithmic differentiation to find the derivative:

$$y = \sqrt{\frac{x^2 + 2}{x^2 - 2}}$$

Question 12 **[4]**

Given the following function:

$$f(x) = (x + 2)^3 - 2$$

12.1 Determine the *domain* of the function. [1]

12.2 Determine $f'(x)$ [1]

12.3 Determine $f''(x)$ [1]

12.4 Determine the interval where the function is concave up. [1]

Question 13 **[9]**

Given the following function:

$$f(x) = \frac{2x^2}{x^2 - 1}$$

13.1 Determine the critical points/s of $f(x)$. [4]

13.2 Determine the $f''(x)$. [3]

13.3 Using the second derivative test, determine whether the critical point/s determined in Question 8.1 are a local minimum or local maximum. [2]

Question 14 [7]

Find the indefinite integral:

14.1 [3]

$$\int \frac{e^x + e^{2x}}{e^x} dx$$

14.2 [4]

$$\int (2x + 2)e^{2x^2+4x+1}\,dx$$

<u>Question 15</u> **[7]**

The demand function for a given product is $p = f(q) = 100 - 0.05q$. The supply function is $p = g(q) = 10 + 0.1q$.

Find the Consumer's Surplus and Producer's Surplus under market equilibrium.

End of Paper – Total 89 Marks

Use this space if you want to redo a question. Clearly indicate at the question that the answer is on Page 14.